Appendix 1

Identification Results

The model parameters including the series resistances are given as follows,

$$r_1 = \frac{n_1 Z_{11b} + Z_{11a}\omega n_2 + \omega n_3 Z_{12a} + Z_{12b}}{\omega n_2} \tag{1}$$

$$r_2 = \frac{n_1 Z_{11b} + Z_{22a}\omega n_2 + \omega n_2 Z_{12a} + Z_{22b}}{\omega n_2} \tag{2}$$

$$g_2 = -\frac{\omega n_2}{(n_1)^2 Z_{11b} + n_1 n_3 Z_{12a}\omega + n_1 Z_{12b} - n_2 Z_{12a}\omega + n_2 n_3 Z_{12b}(\omega)^2 + (n_2)^2 Z_{11b}(\omega)^3} \tag{3}$$

$$g_1 = n_1 g_2 \tag{4}$$

$$c_1 = n_2 g_2 \tag{5}$$

$$c_2 = n_3 g_2$$

$$Z = \frac{\frac{1}{Z_{12}} - g_1 - g_2 - j\omega c_1 - j\omega c_2}{(g_1 + j\omega c_1)(g_2 + j\omega c_2)} \tag{6}$$

where ω is test frequency, Z_{ija} and Z_{ijb}, $i=1,2$, $j=1,2$ are the real and imaginary parts of the Z-parameters respectively, and n_k, $k=1,2,3$ can be calculated from the knowns, whose expressions along with the detailed mathematical manipulations were given in the appendix.

An example

For a simple example to show that the proposed method can solve the problem, let us suppose that

$r_1 = r_2 = 1,$

$g_1 = 2,$

$g_2 = 3,$

$c_1 = 2,$

$c_2 = 4,$

$$Z = \frac{1 - j\omega}{4}$$

For this simple example, the measurements of Z-parameters can be easily simulated by simple calculation as indicated in (7)-(9) in the appendix. They are,

$$\omega = 1,$$

$$Z_{11a} = \frac{377}{328}$$

$$Z_{11b} = \frac{-51}{328}$$

$$Z_{12a} = \frac{2}{41}$$

$$Z_{12b} = \frac{-5}{82}$$

$$Z_{22a} = \frac{45}{41}$$

$$Z_{22b} = \frac{-5}{41}$$

$$\omega_2 = 2$$

$$Z_{11a}^2 = \frac{10145}{9442}$$

$$Z_{11b}^2 = \frac{-711}{4721}$$

$$Z_{12a}^2 = \frac{50}{4721}$$

$$Z_{12b}^2 = \frac{-128}{4721}$$

$$Z_{22a}^2 = \frac{4896}{4721}$$

$$Z_{22b}^2 = \frac{-488}{4721}$$

substitue them to (27) to (35) to calculate coefficients (See appendix 2 for definition) we get

$$a_1 = \frac{116037}{1548488}$$

$$a_2 = \frac{7392}{193561}$$

$$a_3 = \frac{11709}{193561}$$

$$d_1 = \frac{225645}{1548488}$$

$$d_2 = \frac{-124167}{774244}$$

$$d_3 = \frac{-2613}{387122}$$

$$d_5 = \frac{13131}{193561}$$

$$d_5 = \frac{-28842}{193561}$$

By further substituting to (46) to (48) we get

$n_1 = \frac{2}{3}$

$n_2 = \frac{2}{3}$

$n_3 = \frac{4}{3}$

So we can calculate r_1 and r_2 from (50) and (51), the results are

$r_1 = 1$

$r_2 = 1$

and g2 from (52)

$g_2=3$

from (43) to (45)

$g_1=2$

$c_1=2$

$c_2=4$

which are exactly the same as our assumption. For a more practical example, the results' accuracy relies heavily on the precision of the software. It needs to be studied that what precision is needed for our test purpose.

Discussion and conclusions

- In the report, we suppose that two frequencies are used in the measurements. It is noticed that one of them can be zero, that means we can use a DC test and an AC test to identify all the parameters. But in this case, the formulas are more complicated as less information obtained. In fact, a second order, two variables non-linear equation system has to be solved, maybe numerically. Further study is carrying out along this direction hoping find a way to find a relatively simple symbolic solution.
- The calculation can be further reduced as we use two frequencies with one doubles the other, which is the case in the current test system.
- It is possible to locate leakage fault by using the series resistance.
- The line termination parameters, which are represented by a combined impedance Z in our discussion, can be further identified
- It can be concluded that all the parameters in the enhanced line model can be uniquely identified with two frequencies measurements. For the DC and AC case, the line parameter and the combined line termination can be identified, but the individual line termination parameters remain unsolved